Mechanical Eng. Department		Semester: Autumn 2019
Level: 2nd Year		Final Exam: Mathematics IV
Examiner: Dr. Mohamed Eid		Code: Math 201
Time allowed: 2 hours		Date: January 6, 2020

The Exam consists of one page   Answer all questions   No. of questions: 4   Total Mark: 40

Question 1 (12 marks)

(a) Show that the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ is symmetric and find its eigenvalues and

eigenvectors. Also, find its inverse if exists.

(b) Find $f(A) = 2^A$, where $A = \begin{bmatrix} 0 & 1 \\ 6 & 1 \end{bmatrix}$.

(c) Write P in matrix form and determine its type:

(i) $P = z^2 + 3y^2 + 2x^2 - 2xy + 2yz$.   (ii) $P = (2x + y + 3z)^2 - 4xy$.

Question 2 (10 marks)

(a) Find the Laplace transform of the functions:

(i) $f(t) = 2 + e^t + t\cos t$ (ii) $f(t) = t^2 + e^{2t}.\sin 3t$ (iii) $f(t) = (\sin 2t - \cos t)^2$

(b) Find the inverse Laplace transform of the following:

(i) $F(s) = \dfrac{s^2+3}{s^2+4}$ (ii) $F(s) = \dfrac{s+2}{s^2+2s+2}$ (iii) $F(s) = \dfrac{s}{s^2+4}e^{-2s}$

Question 3 (12 marks)

Solve the partial differential equations:

(a) $u_x + u_y = y$ (b) $u_{xy} - u_{yy} = e^{3x-y}$

(c) $u_{xx} - 9u_{yy} = \cos(2x + y)$ (d) $u_{xx} - 2u_{xy} + u_{yy} = x^2y^2$

Question 4 (6 marks)

By Laplace transformations, solve the differential equations:

(a) $y`` - 2y` = e^{2t}$,     $y(0) = 0, \ y`(0) = 1$.

(b) $x\,u_x + u_t = x\,e^{-t}$, $u(x, 0) = 0$.

Good Luck   *Dr. Mohamed Eid*